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ATTENUATION OF LOCAL GAS SWIRLING IN A CHANNEL OF ANNULAR CROSS SECTION

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The attenuation of local gas swirling in smooth channels of annular cross section with r_{in}/r_o = 0.89-0.95 is investigated. Calculating equations satisfactorily describing the experimental data are obtained.

We consider the established flow of a viscous incompressible liquid in a relatively thin cylindrical channel of annular cross section with a ratio of inside to outside radii $r_{in}/r_o \approx 0.9$ behind a swirling device at moderate velocities.

Under the assumption of constancy of all the stream parameters along the circumference of the channel, isotropy of the turbulent properties, the absence of secondary flows, and $\partial^2V/\partial x^2 \ll \partial^2V/\partial r^2$, the equations of conservation of momentum [1], written through the shear stresses, have the form

$$\frac{\partial P}{\partial x} = \frac{1}{r}\frac{\partial (r\tau_{rx})}{\partial r}, \tag{1}$$

$$\rho V_x \frac{\partial V_\theta}{\partial x} = \frac{1}{r^2}\frac{\partial (r^2\tau_{r\theta})}{\partial r}. \tag{2}$$

In connection with the fact that the variation along the channel of the average swirling over the cross section V_θ/V_x is of primary interest for engineering applications, it is expedient to convert to parameters averaged over a cross section in Eqs. (1) and (2).

Taking the relative thickness of the channel as small for the case under consideration, $\delta/r_o \leq 0.1$, where $\delta = r_o - r_{in}$, taking the shear stresses at the inner and outer walls as equal in absolute value and opposite in sign, $\tau_{wo} = -\tau_{win} = -\tau_w$, as well as $\partial/\partial r(\partial p/\partial x) = 0$, we integrate Eqs. (1) and (2) over r from r_{in} to r_o, having preliminarily multiplied the first by r and the second by r^2. After integration we average the terms of the second equation over the channel cross section. As a result, we obtain

$$dP/dx = -2\tau_{wrx}(r_o + r_{in})/(r_o^2 - r_{in}^2) = -2\tau_{wrx}/\delta = -2(\tau_w/\delta)\cos\varphi, \tag{3}$$

$$\rho \int_{r_{in}}^{r_o} r^2 V_x (dV_\theta/dx)\,dr \Big/ \int_{r_{in}}^{r_o} r\,dr = -2\tau_{wr\theta}(r_o^2 + r_{in}^2)/(r_o^2 - r_{in}^2)$$

or, replacing $(r^2_o + r^2_{in})$ by $(r_o + r_{in})^2/2$ with an error of no more than 1%,

$$\rho V_x (dV_\theta/dx) = -2(\tau_w/\delta)\sin\varphi. \tag{4}$$

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In (4) and below V is the velocity averaged over a cross section.

For liquid motion in a channel formed by two parallel walls, under the conditions $|\tau_{wo}| = |\tau_{win}| = \tau_w$ [1],

$$\tau_w = \frac{\xi}{8}\rho V^2. \qquad (5)$$

Substituting Eq. (5) for τ_w into (3) and (4), we obtain the final form of the differential equations for calculating the variation of the swirling V_θ/V_x and the pressure along the channel,

$$\frac{dP}{dx} = -\frac{\xi\rho}{2d_h}V^2\frac{1}{\sqrt{1+(V_\theta/V_x)^2}},$$
$$V_x\frac{dV_\theta}{dx} = -\frac{\xi}{2d_h}V^2\frac{V_\theta/V_x}{\sqrt{1+(V_\theta/V_x)^2}} \qquad (6)$$

or

$$\frac{d(V_\theta/V_x)^2}{(V_\theta/V_x)^2\sqrt{1+(V_\theta/V_x)^2}} = -\frac{\xi}{d_h}dx. \qquad (7)$$

In the case of ξ = const, after integrating Eq. (7) from x_0 to x [x_0 is the coordinate of the cross section with a known swirling $(V_\theta/V_x)_0$], we have

$$V_\theta/V_x = 2\sqrt{A}/(1-A), \qquad (8)$$

where

$$A = [(\sqrt{1+(V_\theta/V_x)_0^2}-1)/(\sqrt{1+(V_\theta/V_x)_0^2}+1)]\exp(-\xi(x-x_0)/d_h).$$

If we take $\xi = B/Re^n = CV^{-n}d^{-n}_h$ (e.g., the Blasius law, valid for smooth channels for $4\cdot10^3 \le Re \le 10^5$ [1]), Eq. (7) takes the form

$$d(V_\theta/V_x)/\{(V_\theta/V_x)[1+(V_\theta/V_x)^2]^{\frac{1-n}{2}}\} = -(\xi_x/2d_h)\,dx, \qquad (9)$$

where ξ_x is the hydraulic resistance coefficient, constant along the channel length, calculated from the velocity $V_x = G/\rho F_x$. In general form the integral on the left side of Eq. (9) cannot be expressed through elementary functions for an arbitrary value of n. Therefore, a dependence of the type $V_\theta/V_x = f[(V_\theta/V_x)_0, \xi_x(x-x_0)/d_h]$ is obtained as a result of numerical integration of the left side of Eq. (9) from $(V_\theta/V_x)_0$ to $(V_\theta/V_x)_x$.

In such a formulation for the case of laminar flow (n = 1) the solution of (9) has the form $V_\theta/V_x = (V_\theta/V_x)_0 \exp[-\xi_x(x-x_0)/2d_h]$.

Integrating Eq. (6) using (8), we obtain

$$P = P_0 - \frac{G^2}{\rho F_x^2}\left\{\frac{x-x_0}{2d_h}\xi + \ln\left[\frac{1-A}{1-A\exp(\xi(x-x_0)/d_h)}\right]\right\}, \qquad (10)$$

where P_0 is the pressure at $x = x_0$.

The data on the attenuation of local gas swirling in channels of annular cross section available in the literature known to the authors [2-4] were obtained for channels having a ratio r_{in}/r_o considerably less than 0.9. In the present work we experimentally determined the characteristics of the attenuation of local swirling in three channels of annular cross section having a ratio r_{in}/r_o = 0.89-0.945. Channel I has an inside diameter d_{in} = 29.8 mm and an outside diameter d_o = 32 mm (r_{in}/r_o = 0.932, d_h = 2.2 mm); II) d_{in} = 34 mm, d_o = 38 mm (r_{in}/r_o = 0.89, d_o = 4 mm); III) d_{in} = 36 mm, d_h = 38 mm (r_{in}/r_o = 0.945, d_o = 2 mm). Air was used as the working substance in all the experiments.

In channel I, the local stream swirling was produced in a short section with multiple-thread spiral finning. The angle φ was determined visually from the position of a hair with a thickness of several hundredths of a millimeter, fastened at one end to the inner surface of a transparent section of the channel wall and blown by the gas stream. The hair was ob-

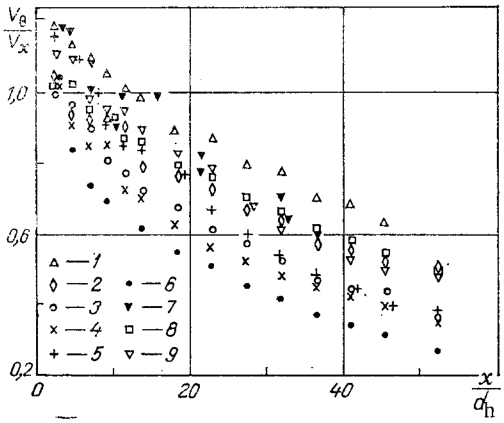

Fig. 1. Attenuation of swirling along the length of an annular channel: 1) $Re_x = 3.8 \cdot 10^4$, h = 20 mm; 2) $3.8 \cdot 10^4$, 10 mm; 3) $9 \cdot 10^3$, 10 mm; 4) $4.7 \cdot 10^3$, 10 mm; 5) $2.5 \cdot 10^3$, 20 mm; 6) $2.5 \cdot 10^3$, 10 mm; 7) $7.2 \cdot 10^4$, $r_{in}/r_o = 0.4$ [3]; 8) $Re_x = 2.2 \cdot 10^4$, h = 10 mm; 9) $9 \cdot 10^3$, 20 mm.

served through a microscope with two mutually perpendicular scales in the field of view, allowing us to determine $\tan \varphi = V_\theta/V_x$ in the form of the ratio of the coordinates of intersection of the image of the hair with these scales. The section of the channel with a transparent wall was illuminated by the bulb of a stroboscopic tachometer. When the flash frequency of the bulb approached the frequency of oscillations of the hair in the gas stream, the image of the latter became visible. The distance between the exit from the finned section and the indicator hair was varied by longitudinal movement of the cylindrical insert forming the inner wall of the channel, with the fins fastened to it.

Since the vortex wakes behind the fins in the gas stream are retained over a certain distance downstream from the finned section, the quantity V_θ/V_x varies periodically along the circumference of the channel at x = const. To measure the quantity V_θ/V_x in the entire range of its variation at a certain fixed distance from the exit from the swirling section, the cylindrical insert with the short fins was rotated about its axis in such a way that the hair was acted on by the required region of the stream.

The experiments were carried out in the range of Reynolds numbers $2.5 \cdot 10^3 \leq Re \leq 3.8 \cdot 10^4$ at the entrance to the swirling device. Here the gas velocity did not exceed 20 m/sec, the pressure was close to 0.1 MPa, and the temperature was 20°C.

To clarify the influence of the length of the swirling device in the axial direction on the characteristics of the initial swirling, experiments were carried out with finned sections with lengths h = 10 mm and 20 mm and a tangent of the angle of inclination of the fins to the channel axis of 1.25, i.e., $(V_\theta/V_x)_0 = 1.25$.

In channels II and III the stream swirling was accomplished through tangential supply of the gas at the channel entrance. The variation of the swirling along the length was determined from the trajectory of drops of a tracer liquid supplied through a capillary tube at a distance of 25 mm downstream from the exit plane of the collector for the tangential gas supply. The trajectory was recorded in the form of the track of the tracer on the inner wall of the channel. The function $V_\theta/V_x = f(x)$ was determined as the dependence on x of the tangent of the angle between the tangent to the corresponding point of the trajectory and the generating line of the cylinder of the inner wall, $\tan \varphi = f(x)$.

The experimental data on the attenuation of swirling in channel I show that at x = const the measured quantity V_θ/V_x is variable along the circumference of the channel, since disturbances in the stream behind the spiral fins are retained over a certain distance from the swirling section. The range of deviation of V_θ/V_x from the mean value varies from 15-20% immediately behind the swirling device to 5-6% at a distance of 50 calibers (x/d_h) downstream, with the main variation occurring over the distance of the first 10-15 calibers. The decrease in this range with greater distance from the swirling section gives an idea of the intensity of dissipation of the wake behind the fins.

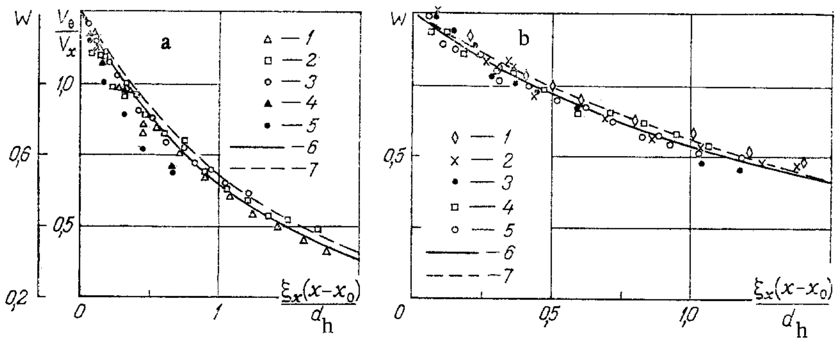

Fig. 2. Comparison of calculated and experimental data on the attenuation of swirling in channel I when the length of the swirling section is 20 mm (a) and 10 mm (b). a: 1) $Re_x = 2.5 \cdot 10^3$, $\xi_x = 0.045$; 2) $9 \cdot 10^3$, 0.0325; 3) $3.8 \cdot 10^4$, 0.0226; 4, 5) $(V_\theta/V_x)_o = 0.87$ and 1.87, respectively; $Re_x = 7.2 \cdot 10^4$; $\xi_x = 0.0194$, $r_{in}/r_o = 0.4$ [3]; 6) calculation from Eq. (8); 7) from Eq. (9); b: 1) $Re_x = 2.5 \cdot 10^3$, $\xi_x = 0.045$, $(V_\theta/V_x)_{oc} = 0.9$; 2) $4.7 \cdot 10^3$, 0.038, 1.02; 3) $9 \cdot 10^3$, 0.033, 1.02; 4) $2.2 \cdot 10^4$, 0.026, 1.1; 5) $3.8 \cdot 10^4$, 0.023, 1.06; 6, 7) calculation from Eqs. (8) and (9), respectively, for $(V_\theta/V_x)_{oc} = 1.1$. $W = (V_\theta/V_x)/(V_\theta/V_x)_o$.

The efficiency of the finned swirling section, by which we understand the imparting to the working substance of uniform swirling, the maximum possible under the conditions of the given geometry, increases with an increase in its length, other conditions being equal (see Fig. 1). This can be explained by the fact that with a decrease in the length of the finned section, when the path traveled by the working substance within the section becomes comparable with the distance between the fins, not all the mass of the working substance acquires the direction of motion corresponding to the angle of inclination of the fins; the stream has a less orderly character and the deleterious actions of local disturbances in flow over the entrance and exit edges of the fins are more fully displayed.

The experimental data on the attenuation of local swirling in channel I with swirling sections 10 mm and 20 mm long (Fig. 1) allow us to conclude that the swirling is attenuated less intensively with an increase in Re. The use of a finned swirling device with $h/d_h = 9.1$ allows one to obtain a value of $V_\theta/V_x = 1.25$ at the exit from the swirling section, i.e., equal to the tangent of the angle of inclination of the fins to the channel axis. A comparison of the experimental data obtained in channel I with the results of calculations of the attenuation of swirling from Eqs. (8) and (9), where $(V_\theta/V_x)_o$ was taken as 1.25, for the case of h = 20 mm ($h/d_h = 9.1$), is shown in Fig. 2a.

As mentioned above, for h = 10 mm ($h/d_h = 4.5$) the swirling at the exit from the finned swirling device is less than the tangent of the angle of inclination of the fins [$(V_\theta/V_x)_o <$ 1.25]. Under these conditions the proposed calculating model allows us to determine the value of $(V_\theta/V_x)_o$ right at the exit from the swirling device, difficult to measure because of the pronounced distrubances of the stream. Here it is assumed that the true value of $(V_\theta/V_x)_o$ corresponds to the calculated value for which the experimental data are most fully described by the proposed calculating relations. In Fig. 2b we present experimental data on the attenuation of swirling downstream from a finned swirling device with a length h = 10 mm ($h/d_h = 4.5$) and the approximating calculating functions obtained from Eqs. (8) and (9) for $(V_\theta/V_x)_{oc} = 1.1$, $(V_\theta/V_x)_{oc}$ is the calculated value of $(V_\theta/V_x)_o$ at the exit from the swirling device for which the experimental data are satisfactorily described by Eq. (8).

Experimental and calculated data on the attenuation of local swirling in channels II and III are presented in Fig. 3. The experimental data were generalized in the dimensionless coordinates V_θ/V_x, $(V_\theta/V_x)/(V_\theta/V_x)_o$, and $(\xi/d_h)(x - x_o)$, which permits the use of the results of the present work for calculations of the attenuation of local swirling in other channels of this type. It should be noted that the complex $(\xi/d_h)(x - x_o)$ appearing in the solutions of Eqs. (6) and (7), as applied to the problem under consideration, is analogous in a certain sense to the Euler number and it characterizes the flow in a channel of annular cross section with a small relative thickness in the section after the swirling device. In this section the circular component of the momentum acquired by a liquid element in the swirling device is expended on overcoming frictional forces in the circular direction. The complex $\xi(x - x_o)/d_h$

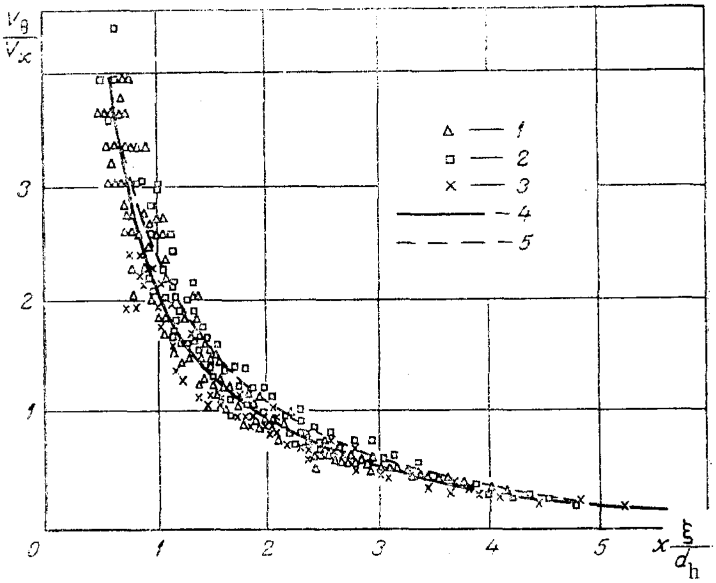

Fig. 3. Comparison of calculated and experimental data on the attenuation of swirling in channels II and III: 1) d_h = 4 mm, $Re_x = 1.18 \cdot 10^4$, $\xi_x = 0.03$; 2) 4, $5.57 \cdot 10^3$, 0.037; 3) 2, $1.58 \cdot 10^4$, 0.028; 4, 5) calculation from Eqs. (8) and (9), respectively, for $(V_\theta/V_x)_0 = 4$ and $\xi x_0/d_h = 0.7$.

characterizes the relative variation of this momentum component along the channel, just as it characterizes the pressure losses in overcoming frictional forces in the direction of grad P.

When the relative thickness of the channel is small, $\delta/r_0 \approx 0.1$, the peculiarities of curved motion of the swirled stream are weakly expressed, and the empirical coefficient of hydraulic resistance for axial flow in the same channel can be used in the analysis of swirled motion using Eqs. (8) and (9). In the case under consideration, this coefficient determines the connection between the frictional forces and the velocity head in both straight and curved flows in qualitatively the same way [5]. Therefore, in calculations of the attenuation of swirling, and in the joint presentation of calculated and experimental data in dimensionless coordinates containing the coefficient ξ, the quantity $\xi(Re)$ was determined from the function obtained experimentally for the given channel under the conditions of axial flow. The experimental data on the hydraulic resistance coefficient obtain for channels I, II, and III with axial flow of the working substance in the range of $5 \cdot 10^3 \leq Re \leq 10^5$ are satisfactorily described by the Blasius function $\xi = 0.3164/Re^{0.25}$ [1], as they should be for smooth channels.

The calculated functions (8) and (9) describe the test data on the attenuation of local swirling in channels of annular cross section with a ratio r_{in}/r_0 = 0.89-0.945 with a satisfactory accuracy for engineering applications (Figs. 1-3). Under the conditions of relatively low initial swirling $(V_\theta/V_x)_0 \leq 2$, Eqs. (8) and (9) are comparable in accuracy. With an increase in the initial swirling to 4, the discrepancy between (8) and (9) increases, with the simpler expression (8), obtained in the approximation of ξ = const, giving a better approximation of the test data than (9). This can be explained by the fact that under the conditions of high initial swirling Eq. (9), obtained in the approximation of $\xi(Re) = B/Re^n$, does not allow for the energy losses, increased in comparison with the case of low swirling, of the stream in the entrance section of the channel, due to the retention of the more intense flow peculiarities acquired by the stream in the swirling device in the given case. The predetermination of the function $\xi(Re)$ in the entrance section of the channel, on the basis of the individual properties of the swirling device, evidently will improve the approximation of experimental data using a calculating function of the type (9).

The satisfactory agreement between the experimental data and the results of the preliminary calculations indicates the possibility of using the proposed calculation model to predict the characteristics of engineering devices with local stream swirling in channels of annular cross section with $r_{in}/r_0 \geq 0.9$.

NOTATION

V, velocity; P, pressure; ρ, density; x, r, θ, axial, radial, and angular cylindrical coordinates; τ_r, shear stress with respect to a surface perpendicular to r in the direction of the velocity vector V; τ_{rx}, $\tau_{r\theta}$, projections of τ_r onto the x axis and the perpendicular to the x, r plane; d_h, hydraulic diameter of the channel; G, mass flow rate; F_x, cross-sectional area of the channel perpendicular to the x axis; ξ, hydraulic resistance coefficient; φ, angle between the vector of the average gas velocity and the x axis; $\zeta_x = f(Re_x)$, the quantity ξ for axial flow.

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LAMINAR FLOWS OF A CONDUCTIVE LIQUID BETWEEN POROUS DISKS IN A TRANSVERSE MAGNETIC FIELD

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We investigate the influence of a transverse magnetic field on the self-similar axisymmetric flows of a viscous conductive liquid between permeable disks.

The flow of a conductive liquid in a plane channel with permeable walls under the influence of a transverse magnetic field has been studied in [1]. A self-similar solution of the problem valid for any Hartmann numbers M was obtained in the form of a regular expansion in powers of a small parameter; the parameter used was the Reynolds number R, calculated on the basis of the velocity of injection or suction. Later studies [2, 3] dealt with the corresponding flows in the case of intensive symmetric bilateral injection, with large negative values of R and small values of the parameter $\mu = M^2/R$. An asymptotic analysis of the effect of the magnetic field on flows in a plane channel that were produced by introducing the conductive liquid through one of the walls and removing it through the other, in nearly asymmetric regimes, was carried out in [4].

The behavior of a conductive liquid that occupies a half-space bounded by a rotating permeable disk (of infinite radius) was studied in [5], where, in particular, it was shown that there exist self-similar flows whose radial velocity component u and axial velocity component w, as in the case of flows of a nonconductive viscous liquid around an impermeable rotating (or motionless) disk, discovered more than 60 years ago by Karman [6], can be represented in the form

$$u = r\omega F(\zeta_*),\ \ w = \sqrt{\nu\omega}\,H(\zeta_*),\ \ \zeta_* = z\sqrt{\omega/\nu}. \tag{1}$$

The flows considered here are caused not by the rotation of the disk but by the suction or injection through the permeable walls. Therefore, the quantity we have used here as the

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